Engineering Topological Superconductivity via Non-uniform Magnetic Textures: A Real-Space Analysis

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Abstract

Using real-space computational methods, we investigate the role of magnetic texture in engineering topological superconducting phases in extended s-wave ($s_{\pm}$-wave) Fe-based superconductors. We compare two configurations: a hybrid superconductor/semiconductor/ferromagnet system with uniform field, and a superconductor/skyrmion lattice where Dzyaloshinskii-Moriya interaction creates a texture acting as effective spin-orbit coupling.

Bulk Chern number calculations reveal a fundamental difference: while uniform ferromagnetic fields produce standard phase diagrams, skyrmion lattices yield entirely new diagrams with higher Chern invariants. Local Chern number analysis shows the skyrmion lattice creates highly inhomogeneous spatial distributions, contrasting the homogeneous ferromagnetic response. This inhomogeneity, reflecting the complex spin structure, drives the emergence of high-invariant phases.

Our results demonstrate nanoscale magnetic texture engineering as a powerful tool for designing novel quantum phases and accessing topological regimes inaccessible via uniform fields.

Introduction

Topological superconductors are among the most active research areas in condensed matter physics due to their unique quantum properties and potential for topological quantum computing [2, 5]. These systems host topologically protected edge states such as Majorana fermions, providing a platform for noise-robust quantum qubits [5]. Inducing topological phases in superconductors has been primarily achieved through uniform magnetic fields and spin-orbit coupling in hybrid superconductor/semiconductor/ferromagnet structures [5, 6]. These standard approaches typically yield phase diagrams with low topological invariants.

However, recent advances in nanoscale magnetic texture engineering, particularly skyrmion structures, have opened new possibilities for controlling quantum phases [3]. Skyrmions are topological spin structures with spatially inhomogeneous textures that differ qualitatively from uniform magnetic fields. The continuous spatial variation of spin directions creates steep gradients that not only produce distinct magnetic properties but also act as effective spin-orbit coupling [3]. This dual functionality, arising from the intrinsic Dzyaloshinskii-Moriya interaction, offers potential for novel topological phases in superconductors.

Analyzing skyrmion lattices and comparing them with uniform magnetic configurations requires real-space computational methods, as the broken translational symmetry limits the applicability of standard momentum-space approaches [3, 1].

In this research, we investigate the effect of magnetic textures on topological superconducting phases using real-space methods. We compare the topological response of an extended s-wave (s±-wave) Fe-based superconductor under two configurations: a hybrid superconductor/semiconductor/ferromagnet system with uniform field, and a superconductor/skyrmion lattice system with periodic inhomogeneous texture [4]. Through bulk and local Chern number calculations, we evaluate the role of magnetic texture in engineering topological states and identify the physical mechanisms underlying the emergence of phases with higher topological invariants [3, 1].

Numerical model and method

In this study, we investigate a hybrid system composed of an unconventional iron-based superconductor, a layer with strong spin–orbit coupling, and a magnetic layer. The key feature of this hybrid system is that the iron-based superconductor induces an *s*-wave pairing in the middle layer through the proximity effect. Moreover, the magnetic layer induces a Zeeman interaction, which breaks the time-reversal symmetry in the middle layer. The simultaneous presence of superconducting, magnetic, and spin–orbit coupling components in the middle layer leads to the emergence of topological superconductivity in this system.The effective Hamiltonian reads:

$$H = -\frac{1}{2}\sum_{i,j,\sigma}\left(t_{ij}c^\dagger_{j\sigma}c_{i\sigma} + \Delta_{ij}c^\dagger_{i\sigma}c^\dagger_{j\bar{\sigma}}\right) - \lambda\sum_{i,\eta=\pm}\eta\, c^\dagger_{i\uparrow}\left(c_{i-\eta\hat{x},\downarrow} - ic_{i-\eta\hat{y},\downarrow}\right) + \text{H.c.} + J\sum_{i,\alpha,\beta}c^\dagger_{i\alpha}\left(\vec{S}_i\cdot\vec{\sigma}_{\alpha\beta}\right)c_{i\beta}.$$

Here, c and $c^\dagger$ are fermionic operators, $i=(i_x,i_y)$ denotes lattice sites on a square lattice. The on-site parameters are $t_{ii}=\mu$ (chemical potential) and $\Delta_{ii}=\Delta_0$ (on-site pairing), while nearest-neighbor parameters are $t_{\langle i,j\rangle}=t$ and $\Delta_{\langle i,j\rangle}=\Delta_1$. The coexistence of Δ_0 and Δ_1 yields extended *s*-wave superconductivity. The parameters λ and J represent the spin–orbit coupling and Zeeman interaction strengths, respectively.

The topological phases are characterized by the Chern number. In systems lacking translational symmetry, we use the real-space formula:

$$C = \frac{1}{2\pi i}\,\mathrm{Tr}\left(P[\partial_x P, \partial_y P]\right),$$

where

$$P = \sum_{n\in\text{Occupied}} |\psi_n\rangle\langle\psi_n|$$

projects onto occupied states. The derivatives are computed via a Fourier-based method:

$$\partial_w P = \sum_{m=-Q}^{Q} c_m e^{-2\pi i m\hat{w}/N_w} P\, e^{2\pi i m\hat{w}/N_w}, \qquad w = x, y,$$

with coefficients c_m obtained by solving $Ac=b$ where $A_{ij}=2j^{2i-1}$ and $b_i=\delta_{i1}$ [3]. Numerical calculations were performed using the TBTK library on a square lattice with periodic boundary conditions, $N_x=N_y=25$, $\Delta_0=0.8t$, $\lambda=0.8t$, and a skyrmion radius of $R=3$.

Results and Discussion

We examine topological phase diagrams as a function of chemical potential μ and Zeeman coupling J for both magnetic configurations on a 25×25 lattice with parameters $\Delta_0=0.8t$, $\lambda=0.8t$ (for ferromagnetic case; $\lambda=0$ for skyrmion).

Figure 1 compares phase diagrams for conventional s-wave pairing ($\Delta_1=0$). Panel (a) shows the uniform ferromagnet case, exhibiting well-defined topological phases with Chern numbers up to $|C|=2$, characterized by sharp phase boundaries and uniform coloring within each region. Panel (b) displays the skyrmion lattice ($R=3$), revealing a markedly different topology: phase boundaries become less distinct, regions show color gradients rather than uniform values, and Chern numbers reach higher values approaching $|C|=3$ in some areas. This contrast highlights how the skyrmion's spatially varying spin texture creates a more complex, inhomogeneous topological landscape compared to the clean transitions of uniform magnetization.

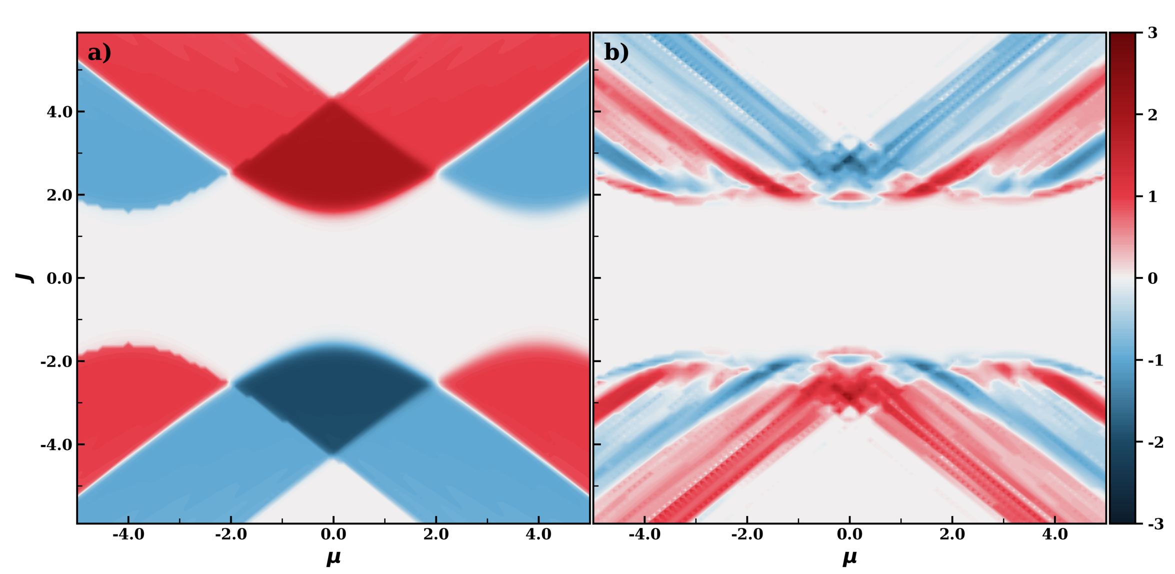

Figure 1. Topological phase diagrams for s-wave pairing ($\Delta_1=0$): (a) uniform ferromagnetic configuration vs. (b) skyrmion lattice ($R=3$).

Figure 2 maps the local Chern number distribution at the point ($\mu=0$, $J=2t$). The ferromagnetic configuration (a) shows spatially homogeneous topology, consistent with translational symmetry. In stark contrast, the skyrmion lattice (b) exhibits strongly inhomogeneous patterns with alternating positive and negative regions forming a periodic structure. This spatial modulation directly reflects the skyrmion's vortex-like spin texture and demonstrates how magnetic inhomogeneity creates localized topological features.

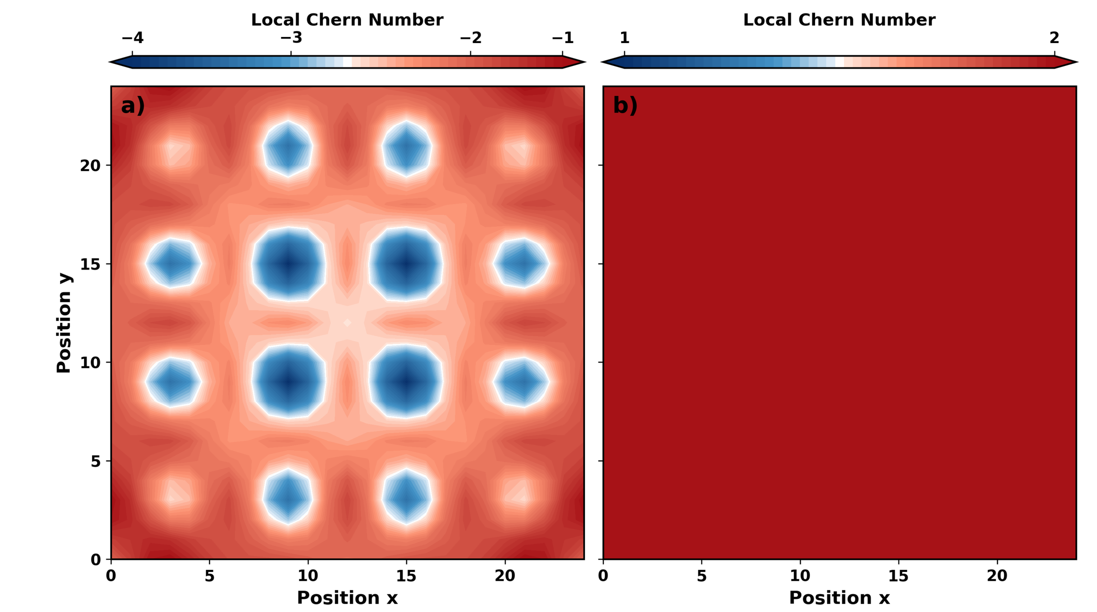

Figure 2. Spatial distribution of the local Chern number at $\mu=0$, $J=2t$: (a) ferromagnetic configuration with homogeneous topology vs. (b) skyrmion lattice with strong inhomogeneity.

Figure 3 explores the extended s-wave regime ($\Delta_1=0.2t$), where nearest-neighbor pairing becomes significant. Panel (a) shows the ferromagnetic case maintains relatively clean phase boundaries similar to conventional s-wave, though with modified topological regions. Panel (b) reveals the skyrmion lattice exhibits even more complex behavior: the phase diagram becomes highly fragmented with intricate stripe-like patterns and numerous small topological domains. This dramatic enhancement of inhomogeneity demonstrates that extended pairing amplifies the texture-driven topological complexity, creating a rich landscape of intertwined phases inaccessible in simpler pairing scenarios.

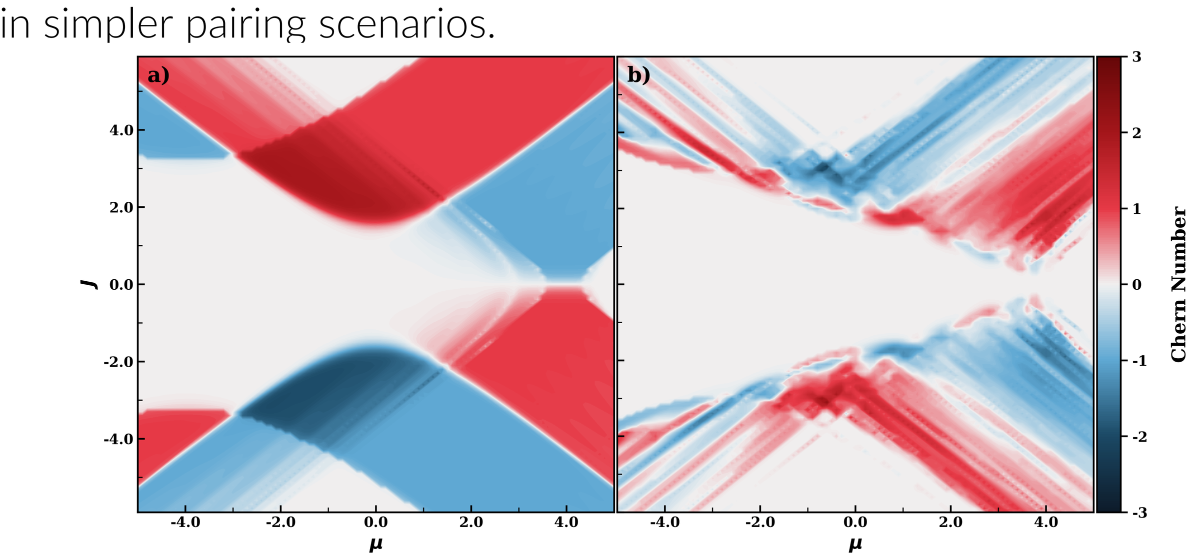

Figure 3. Extended s-wave phase diagrams ($\Delta_1=0.2t$): (a) ferromagnetic configuration vs. (b) skyrmion.

Conclusion

Using real-space methods, we show that skyrmion lattices generate topological phases fundamentally different from uniform ferromagnets. Skyrmions produce higher Chern numbers (approaching $|C|=3$ versus $|C|\le 2$) and highly inhomogeneous spatial distributions, while ferromagnets yield clean phase diagrams with sharp boundaries. This inhomogeneity, arising from the skyrmion's complex spin structure, enables access to novel topological regimes. Extended s-wave pairing further enhances these effects. Our results demonstrate that nanoscale magnetic texture engineering offers new pathways for designing exotic quantum phases in superconductors.

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