Mf-Toolkit: A Python Library for Efficient Multifractal Analysis

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Introduction

Multifractal Detrended Fluctuation Analysis (MFDFA) is a vital tool for revealing hidden scaling complexity in non-linear time series. However, its high computational cost remains a major barrier for large-scale applications. We introduce **mf-toolkit**, an open-source library that solves this issue by using CPU-based parallelization and Numba acceleration to achieve highly efficient MFDFA. To demonstrate its robustness and performance, we apply MF-toolkit to analyze the complex, non-stationary acoustic patterns found in Humpback Whale vocalizations, showcasing its utility for accelerating physics research and advanced signal characterization.

Data and Methods

Data: The time series analyzed in this study is a bioacoustic recording of Humpback Whale (Megaptera novaeangliae) vocalizations. The data was sourced from the publicly available Watkins Marine Mammal Sound Database. The raw audio signal was processed to extract a single time series suitable for multifractal analysis.

Multifractal Detrended Fluctuation Analysis (MFDFA): The core of the analysis, performed using the accelerated, parallelized MFDFA implementation within the toolkit. This implementation leverages Numba for just-in-time compilation and supports multi-core CPU computation for enhanced efficiency.

Crossover Detection: To accurately segment the scaling behavior of the complex signal, we employed two automated crossover detection methodologies available in the toolkit: *Crossover Detection based on Variance of slopes differences (CDV-A)* used for robust identification of a single crossover point; and *Sequential Permutation for Identifying Crossovers (SPIC)* utilized to test distinct scaling regimes within the time series.

Identifying Source: To discern the origin of the multifractality, we can generate surrogate series using two complementary techniques: *Random Shuffling* applied to destroy all temporal correlations while preserving the original signal's probability distribution function (PDF); and *Iterated Amplitude Adjusted Fourier Transform (IAAFT) Shuffling* used to destroy nonlinear correlations while preserving both the PDF and the linear correlation structure.

References

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Detecting Multifractality in Time Series

Utilizing the parallelized MFDFA function within the library, we processed the audio data to characterize its multifractal properties. The analysis successfully extracted all characteristic parameters ($F_q(s)$, $h(q)$, $\tau(q)$, $D(\alpha)$) and can be performed with an internal theoretical validation filter (VALIDATE=True) activated, ensuring the numerical consistency of the calculated exponents before final visualization.

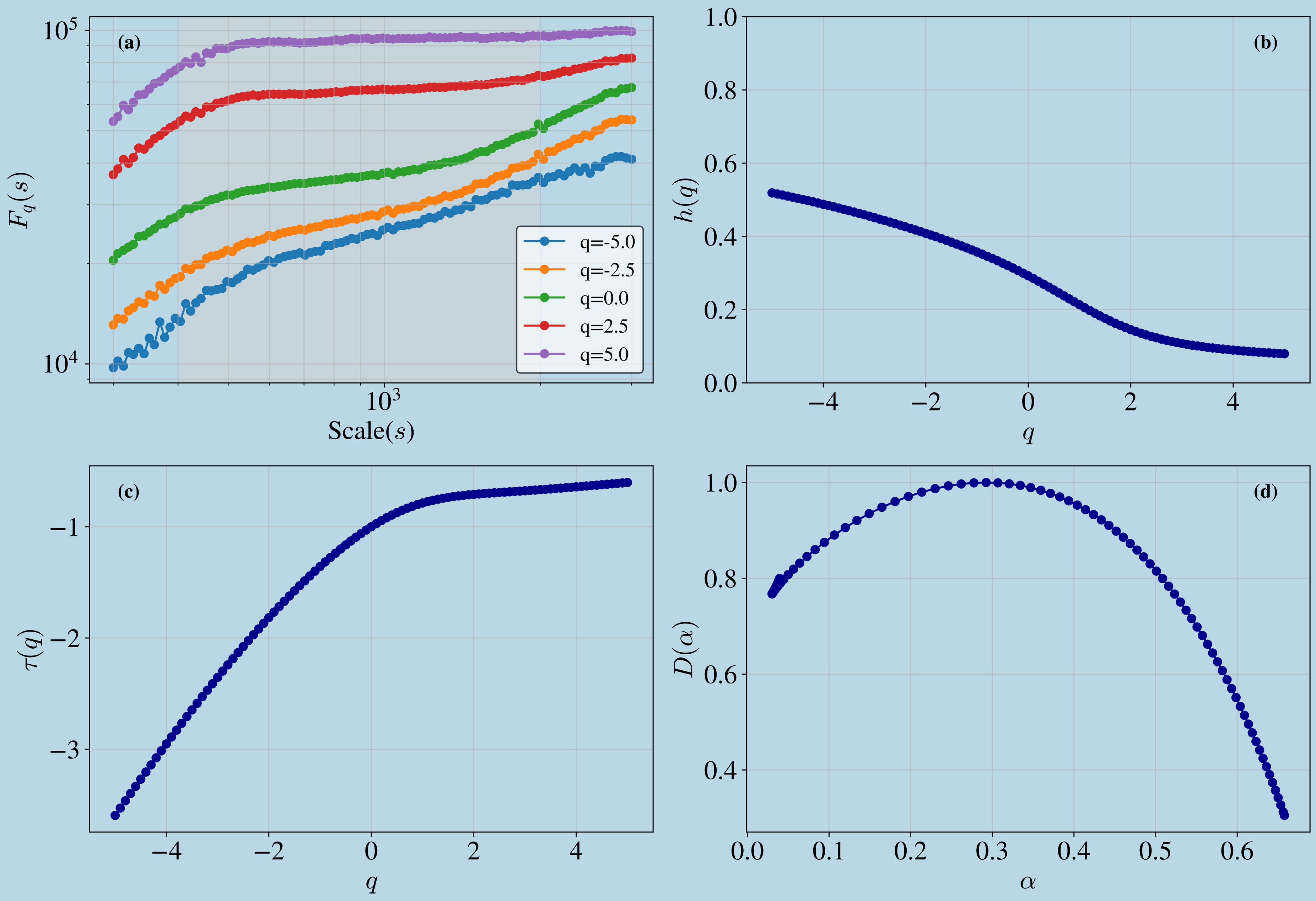

Figure 1: The four panels display the core results from the MFDFA applied to the analyzed time series. Panel (a) shows the generalized fluctuation functions, $F_q(s)$, across various statistical moments q. Panel (b) presents the generalized Hurst exponent, $h(q)$. Panel (c) illustrates the mass exponent, $\tau(q)$. Finally, Panel (d) displays the multifractal singularity spectrum, $D(\alpha)$.

Crossover Detection and Source Identification

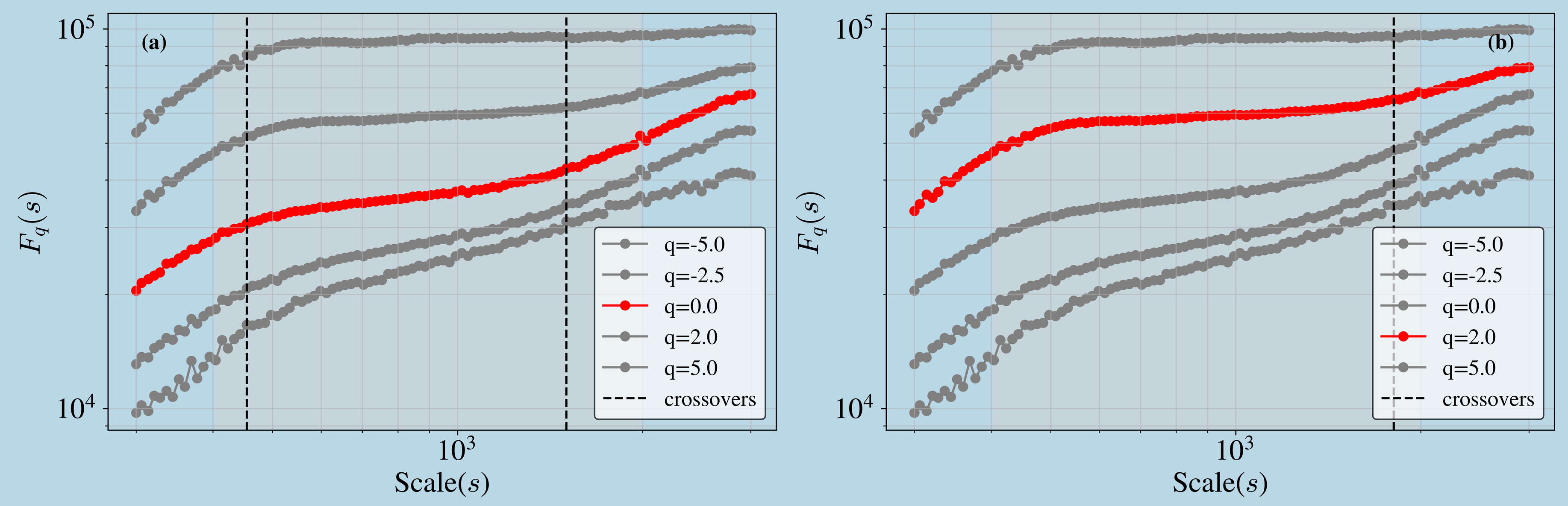

Figure 2: The figure illustrates the application of two distinct methodologies for automatically identifying scaling transitions in $F_q(s)$. Panel (a) displays the results using the SPIC method, which successfully detects two separate crossovers for the $q = 0$ moment. Panel (b) shows the detection result using the CDV-A method for the $q = 2$ moment, demonstrating its capacity to pinpoint only a single crossover point.

The library's automated detection features were validated successfully: the CDV-A method identified one crossover point, while the more sensitive SPIC method successfully detected two distinct crossovers. Furthermore, the multifractality was definitively linked to long-range correlations: applying MFDFA to a randomly shuffled surrogate series resulted in the complete vanishing of multifractality, yielding a constant $h(q) \approx 0.5$. This conclusion is supported by the Q-Q plot of the original series, which confirms that the Probability Distribution Function (PDF) is not heavy-tailed, ruling out value distribution as the primary source.

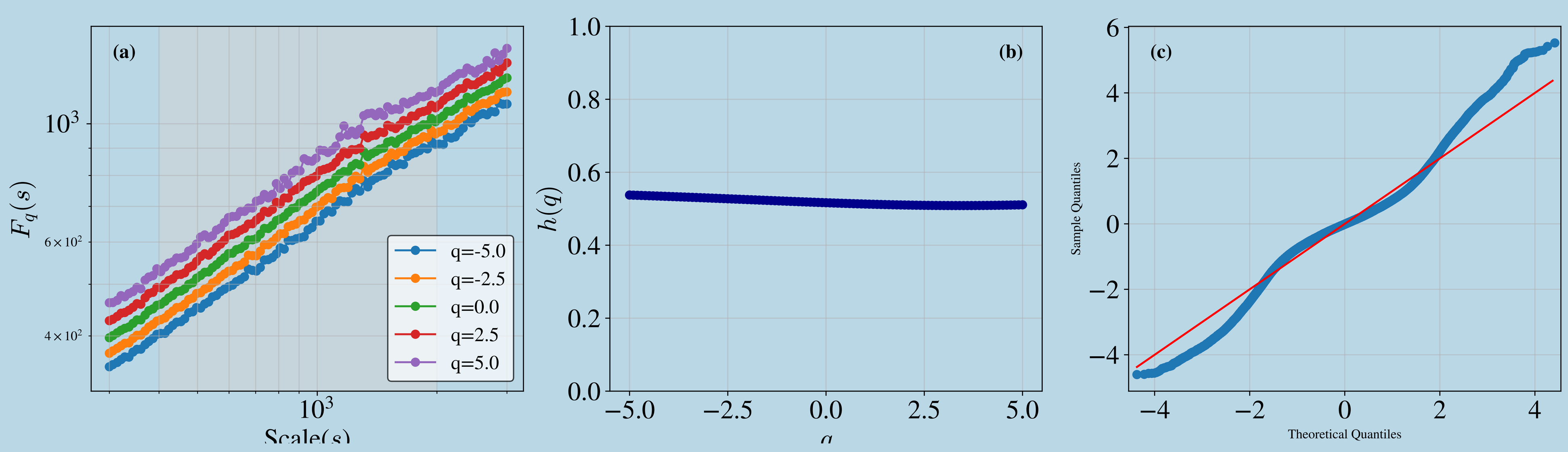

Figure 3: This figure displays the results of the MFDFA applied to a randomly shuffled surrogate time series. Panel (a) shows the generalized fluctuation functions, $F_q(s)$. Panel (b) presents the generalized Hurst exponents, $h(q)$, as a function of the moment exponent q. Panel (c) displays the Quantile-Quantile (Q-Q) plot of the original series, confirming its near-Gaussian value distribution.